Effect of Antenna Performance on the GPS Signal Accuracy

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BIOGRAPHY

Waldemar Kunysz received a Master of Nautical Science Degree from the Merchant Marine Academy, Gdynia, Poland in 1981 and a BSEE from the Technical University of Nova Scotia in 1989. From 1989 to 1995 he specialized in Microwave Landing System phased-array antenna design for Micronav Int. Inc. He is currently with NovAtel Inc. as a RF engineer. He has authored four papers in various conference proceedings. His current research interests are in microwave engineering, electromagnetic propagation, phased-array antenna theory and design, and multipath mitigation techniques.

ABSTRACT

The effect of antenna performance on the carrier phase measurement accuracy is described. The antenna performance characteristics in the presence of multipath are analyzed in terms of an antenna Axial Ratio and an antenna Noise Figure. In this analysis, the multipath is assumed to originate from a single point located on a perfectly conducting ground.

The analysis focuses on the carrier phase sensitivity with respect to various parameters such as antenna Axial Ratio, Noise Figure, ground reflection coefficient, etc. The analytical simulations show that Axial Ratio has significantly more influence on the carrier phase accuracy in presence of multipath than antenna Noise Figure.

I INTRODUCTION

The antenna is an important part of the GPS receiver system. The GPS antenna receives and translates the GPS signal from an electromagnetic wave into a RF signal that contains the amplitude and phase information of the GPS signal. An antenna's characteristics and performance will set the boundaries of how well the GPS receiver system will perform. There is ongoing development of various correction techniques (DGPS, Pseudolites, etc.) to meet an increasing demand for accuracy, integrity, availability and continuity of satellite based positioning. Multipath and receiver system thermal noise still remains as the main obstacles to achieve the above mentioned goals. Some methods are more effective against the thermal noise but less effective against multipath and vice-versa.

The antenna is a natural receptor of multipath generated copies of the GPS signal, therefore the antenna's ability to reject these multipath signals is described in terms of Axial Ratio patterns.

An active GPS antenna (with built in Low Noise Amplifier – LNA) sets the overall Noise Figure of the receiver system, hence the contribution of its Noise Figure on the GPS signal accuracy is also described.

Group delay versus frequency is of most interest in microwave components, while in the case of an antenna it is useful to consider the variation of group delay versus elevation and azimuth angles. Thus, in GPS applications, maximum accuracy requires that the receiving antenna have a uniform group delay response for all angles of incidence in most of the upper hemisphere, otherwise signals from satellites at different elevation/azimuth angles will experience timing errors.[3]

The amplitude and phase of the received RF carrier signal are used as quality measures of the GPS signal accuracy. All modern GPS receivers use carrier phase measurements to achieve sub-centimetre accuracies, which isn't the case with amplitude of the signal. Traditionally, the amplitude of the signal had found little use in GPS signal processing. Recently, a lot of research has been done on using amplitude information in multipath mitigation techniques using "signal-to-noise ratio"(SNR) information [1] [2].

This analysis is performed to determine the limits of GPS signal accuracy and their sensitivity to the following type parameters:

- Axial Ratio of the GPS antenna
- Distance difference between the reflected and direct line-of-sight (LOS) received GPS signal
- Reflection coefficient of the ground plane from which the specular multipath is originated
- Overall Noise Figure of the antenna

- Group Delay variations versus elevation/azimuth angle

II GPS ANTENNA MODEL

Since the GPS antenna must receive a right hand circular polarized (RHCP) signal, it is convenient to define the electric field at any point in space using RHCP and left-hand circular polarized (LHCP) field components as:

$$E = \mathbf{r}E_{RHCP} + \mathbf{l}E_{LHCP}$$

$$r = \frac{1}{\sqrt{2}}\left(\bar{\theta} - j\bar{\varphi}\right) \quad \text{and} \quad l = \frac{1}{\sqrt{2}}\left(\bar{\theta} + j\bar{\varphi}\right)$$

where:
E_θ and E_ϕ are the linear field components in the θ and ϕ directions defined in conventional spherical co-ordinates.

$$E_{RHCP} = \frac{1}{\sqrt{2}}\left(E\theta + jE\varphi\right) \text{ and } E_{LHCP} = \frac{1}{\sqrt{2}}\left(E\theta - jE\varphi\right)$$

Since the Axial Ratio (AR) can be defined as the ratio of major and minor axes of the polarization ellipse, therefore:

$$AR = \frac{E_{maj}}{E_{min}} = \frac{\left|E_{RHCP} + E_{LHCP}\right|}{\left|E_{RHCP} - E_{LHCP}\right|} \qquad (1)$$

II.1 Assumptions

In this study, the GPS antenna is modelled as a point source with a combination of an RHCP and an LHCP pattern whose ratio is defined by the Axial Ratio (AR) of the antenna at given spherical co-ordinates θ and ϕ. The antenna is assumed to be located on the perfectly conducting ground plane. For simplicity of analysis, multipath is assumed to be a specular type reflection originating from a single point located on a perfectly conducting ground. The resulting reflected GPS signal could be modelled as a delayed replica of the line-of-sight (LOS) signal, only with a different phase and amplitude. It is also assumed that the full bandwidth of the GPS antenna is 20 MHz.

II.2 Signal Model

The antenna element has the $\frac{ae^{-jkR}}{R}$ dependence of a spherical wave multiplied by a vector function of angle P (θ,ϕ), called the element radiation pattern. The signal received form a specified location (R, θ, ϕ) can be obtained by superimposing the direct signal and reflected signal. The received carrier signal can be modelled by:

$$E = \frac{ae^{-j\left(k\cdot D_d + \phi_a\right)}}{D_d}\cdot P_a + \frac{ae^{-j\left(k\cdot D_r + \phi_a\right)}}{D_r}\cdot P_b \cdot r \cdot e^{-j\cdot\phi_r} \quad (2)$$

where:

k	-	Wave number ($2\pi/\lambda$)
a	-	Amplitude of LOS signal
P_a	-	RHCP radiation pattern gain
P_b	-	LHCP radiation pattern gain
D_d, D_r	-	Distance of the LOS signal and reflected signal from the satellite to the antenna
ϕ_a	-	Initial phase of the LOS signal
r, ϕ_r	-	Ground reflection coefficient and phase change of the reflected ray due to the reflector.

By rearranging equation (1) we'll get the following relationship:

$$E = \frac{ae^{-j\left(k\cdot D_d + \phi_a\right)}}{D_d}\cdot P_a\left(1 + \frac{(AR-1)\cdot r\cdot e^{-j\left(k\cdot\Delta D + \phi_t\right)}}{(AR+1)\cdot(1+\Delta D)}\right) \quad (3)$$

where:

$$\frac{AR-1}{AR+1} = \frac{P_b}{P_a}$$

ΔD	-	Distance difference between LOS and reflected signal
ϕ_t	-	Total phase of the reflected signal that includes the effect of phase change during reflection from the ground and phase change corresponding to an extra length (ΔD) travelled by the reflected ray with respect to the LOS signal.

III ANALYSIS

The second term in the brackets of equation (3) represents an error term due to multipath effect, while the first term (integer of one) represents the ideal case of receiving a direct signal only. The phase carrier error can be determined by solving for the phase argument of complex quantity E for both cases – with and without multipath effect, and then subtracting them from each other (4).

$$\phi_{error} = \frac{arg(\,E(\,AR,\Delta D, r, \phi_t\,)) - arg(1,0,0,0\,)}{-k} \quad (4)$$

Note that for Axial Ratio = 1 the antenna has a perfect RHCP pattern and therefore an infinite rejection of the LHCP signal. We have four variables, so to determine the sensitivity of the carrier phase two variables are held constant while the other two are varied.

III.1 Carrier Phase Error versus Phase & Distance Difference

Let the axial ratio to be set to 6 dB (AR=1.4125) and let the reflection coefficient to be set to -6 dB. The distance difference between the reflected and LOS signal is varied from 0 to 60 cm and the phase difference from 0 to 5π.

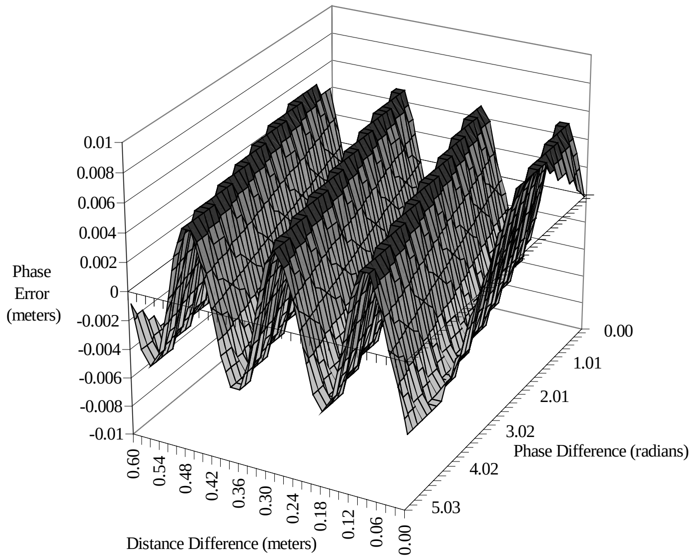

Figure 1 Carrier Phase Error versus distance and phase difference between the direct and reflected signals

The carrier phase error exhibits as expected a sinusoidal change with respect to both parameters – the path (distance) and phase difference between the direct and reflected signals. The magnitude of the carrier phase error is in order of millimetre range.

III.2 Carrier Phase Error versus Axial Ratio (AR) and Ground Reflection coefficient (r)

Let the distance difference, Δ, be set equal to 0.1m and let the phase difference, ϕ_t, be set to 0°. The Axial Ratio is varied from 0 to 30 dB and the ground reflection coefficient is varied from 0 to –100 dB.

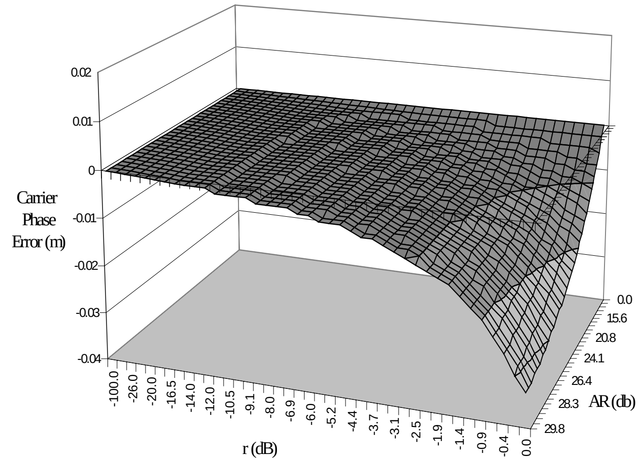

Figure 2 Carrier Phase Error versus Axial Ratio and ground reflection coefficient.

As expected the carrier phase error peaks (-0.035 meter) at the maximum values of Axial Ratio and ground reflection coefficient. The negative sign of the carrier phase error is contributed to an arbitrary setting of variables Δ and ϕ_t.

III.3 Amplitude Error versus Phase & Distance Difference

The amplitude error can be determined by solving for the magnitude argument of complex quantity E for both cases – with and without multipath effect, dividing them, and then computing the log value (5).

$$A_{error} = 20 \cdot log\left(\frac{|E(AR, \Delta D, r, \phi_t)|}{|E(1,0,0,0)|}\right) \qquad (5)$$

Let the Axial Ratio be set to 6 dB (AR=1.4125) and the reflection coefficient be set to 0 dB (r=1). The distance difference between the reflected and LOS signal is varied from 0 to 60 cm and phase difference from 0 to 5π.

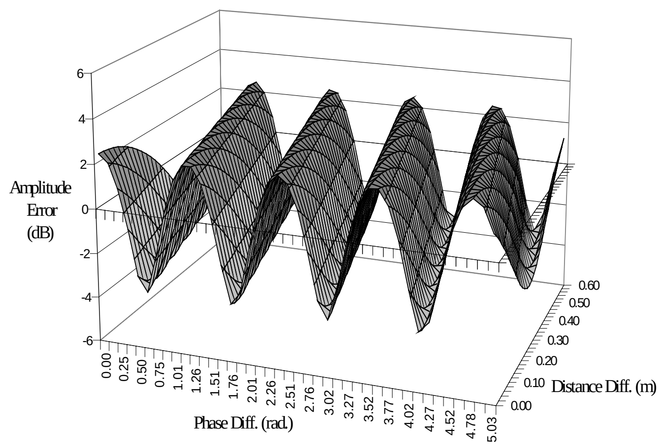

Figure 3 Amplitude Error versus distance and phase difference between the direct and reflected signals

The amplitude error experiences the same sinusoidal pattern change as the carrier phase error, due to the fact that the LOS and reflected signal add to each other in and out of phase depending on the location of the reflection point and its phase shift characteristics.

III.4 Amplitude Error versus Axial Ratio (AR) and Ground Reflection coefficient (r)

Let the distance difference, Δ, be set again to 0.1m and the phase difference, ϕ_t, be set to 0°. The Axial Ratio is varied from 0 to 30 dB and the ground reflection coefficient is varied from 0 to –100 dB.

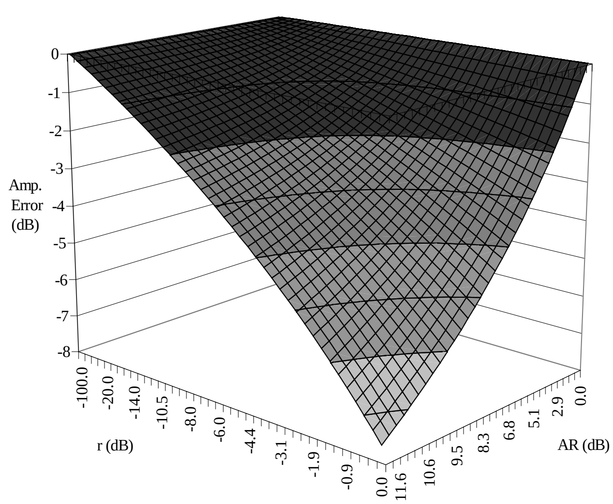

Figure 4 Amplitude Error versus Axial Ratio and ground reflection coefficient

The negative sign of the amplitude error is contributed to an arbitrary setting of variables Δ and ϕ_t. Note that the maximum error occurs, as expected, at the maximum Axial Ratio and ground reflection coefficient conditions.

IV NOISY SYSTEM ANALYSIS

Equation (3) represents an ideal case of a noiseless antenna. Whether the GPS antenna is passive or active (built-in Low Noise Amplifier – LNA) we can assume that the noise of the overall GPS receiver system is introduced in the antenna port. The maximum amplitude of the noise is bounded therefore, by the overall Noise Temperature (T_{eq}) of the receiver system plus the average noise temperature of the sky (100°K). The noise is assumed to have a gaussian phase and amplitude distribution. The Equivalent Noise Temperature of the system is given by:

$$T_{eq} = 100 + 290 \cdot (\text{Noise\_Factor} - 1) \qquad (6)$$

and the noise power is given (assuming we are dealing with Gaussian distribution with zero mean) by

$$|n| = K \cdot T_{eq} \cdot B \qquad (7)$$

where:

K	-	Boltzman constant
B	-	Antenna signal bandwidth

By adding the thermal noise to the signal received by the antenna, see equation (2), we obtain

$$E(n) = E + n \qquad (8)$$

where n is also a complex quantity with random phase and amplitude. To assess the contribution of thermal noise to the error budget, we will remove the multipath component by setting the Axial Ratio to 0 dB (AR=1). The typical range of the Noise Figure can vary anywhere between 1.5 dB to 15 dB. The Noise Figure of the receiver system therefore is varied from 0 to 15 dB, and the noise phase angle from 0 to 6π (random process).

IV.1 Carrier Phase Error versus Noise Figure and Noise Phase Angle

The carrier phase error can be determined, again by solving for the phase argument of complex quantity E for both cases – with and without noise and then subtracting them from each other (3).

$$\phi_{noise} = \frac{\arg(E(n)) - \arg(E)}{-k} \qquad (9)$$

Note that for Axial Ratio = 1 the antenna has a perfect RHCP pattern and therefore an infinite rejection of the LHCP signal. We have four variables, so to determine the sensitivity of the carrier phase two variables are held constant while the other two are varied.

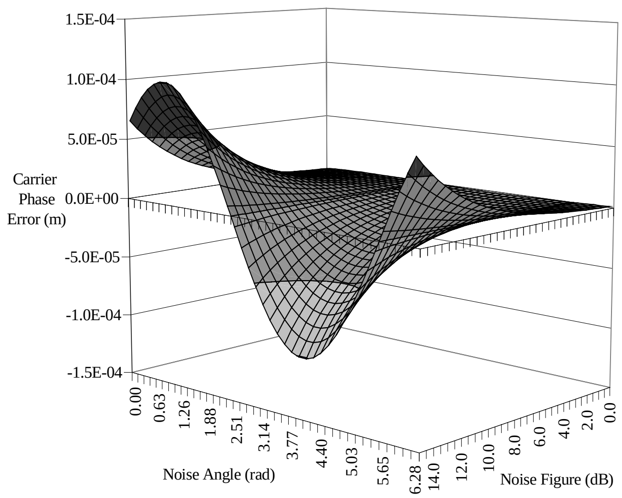

Figure 5 Carrier Phase Error versus Noise Figure and Noise Phase Angle

It is worthy to note that the noise contribution to carrier phase error is on the order of two magnitudes smaller than the error induced from imperfection of the antenna pattern (Axial Ratio) in the multipath environment. In addition, there is not much difference whether the GPS receiver system has the Noise Figure of 1 dB or 6 dB from the carrier accuracy point of view. A higher Noise Figure does, however, affect other system parameters such as the dynamic range of the GPS signal. In the past, a lot of attention was paid to the Noise Figure of the antenna instead to the Axial Ratio.

IV.2 Amplitude Error versus Noise Figure and Noise Phase Angle

The amplitude error can be determined by solving for the magnitude argument of the complex quantity ratio of E (n) and E (without multipath effect), and then computing logarithmic value (10).

$$A_{error} = 20 \cdot \log\left(\frac{|E(n)|}{|E|}\right) \qquad (10)$$

The Noise Figure is varied again between 0 dB and 15 dB and the noise angle between 0 and 6π. See Figure 6.

Again, the thermal noise contribution to amplitude error is on the order of two magnitudes smaller than the error induced from the imperfection of the antenna pattern (Axial Ratio) in the multipath environment. And similarly, there is not much difference whether the GPS receiver system has the Noise Figure of 1 dB or 6.

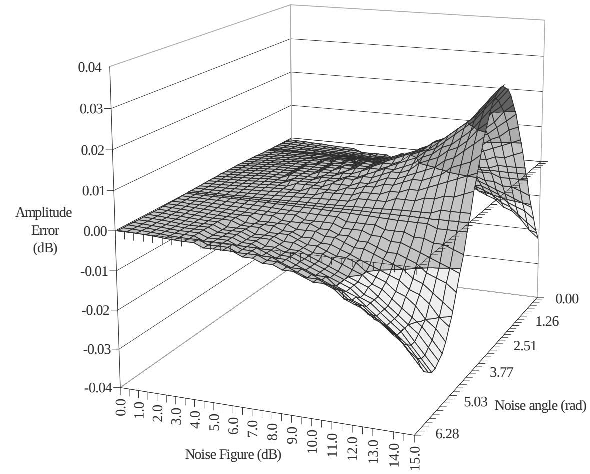

Figure 6 Amplitude Error versus Noise Figure and phase angle.

These results were obtained with a nominal C/No ratio of 50 dB-Hz. For satellites with lower C/No ratio (i.e. 30 dB-Hz) a similar analysis yields the amplitude and carrier phase errors in the range of 0.2 dB and 0.001 meter.

V GROUP DELAY

Group delay variations between two given signals received from two satellites will be a function of phase difference variation of the antenna radiation pattern versus frequency change at the incidence azimuth and elevation angle. Group delay is defined as the derivative of the transfer phase response $\phi(\omega)$ versus frequency ω

$$Group\_delay = \frac{d\phi(\omega)}{d\omega} \qquad (11)$$

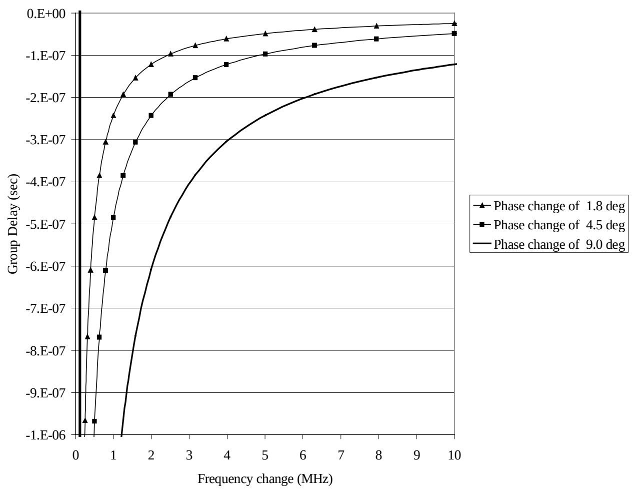

Figure 7 Group Delay variations

Figure 7 is a computational example of three curves where uniform phase differences of 1.8°, 4.5° and 9.0° between two points in the radiation pattern are established across the GPS bandwidth. The group delay variation induced by the radiation pattern of the antenna appears to be a very important parameter in the overall receiver system performance, since it can induce relatively large timing errors.

VI CONCLUSION

A theoretical study of the antenna performance on the GPS signal carrier has been performed in the presence of multipath. Three important aspects of the antenna performance have been analyzed with respect to GPS signal accuracy: the Axial Ratio, Noise Figure and group delay.

Important relationships between carrier phase/amplitude accuracy of the GPS signal and antenna performance parameters have been investigated: these can be summarized by the following assertions:

- Poor Axial Ratio of the antenna pattern will cause significant phase/amplitude errors in a high multipath environment, exceeding by far the thermal noise contribution to the phase/amplitude errors of the GPS signal

- Amplitude error is much more sensitive to multipath than the carrier phase error. This effect can be minimized with an antenna with a low Axial Ratio characteristics, however, on the other hand, the amplitude variation can be used to detect the presence of multipath generated signals.

- Group Delay variations between different points in the radiation pattern of the antenna must be kept to minimum in order to minimize the timing and positioning errors. It is not possible to provide an ideal phase characteristic for practical antennas, but certain antenna designs have much better characteristics than others do.

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